Predicting Noise and Resolution Properties in Tomosynthesis with Statistical Image Reconstruction

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Acknowledgements

The I-STAR Laboratory
Imaging for Surgery, Therapy, and Radiology
• JH Siewerdsen, W Zbijewski
• Y Otake, J Lee
• S Schafer, P deJean
• GJ Gang, P Prakash
• DJ Mirota, A Uneri, S Nithiananthan
• J Yoo, S Reaungamornrat, Y Ding

Hopkins Collaborators
• JA Carrino, A Machado
• RJ Taylor, J Prince, G Hager
• D Reh, G Gallia, J Khanna

Funding Support
• NIH R01-CA112163
Motivation

- Analysis of noise propagation
  - System design (focal spot, dose, etc.)
  - Performance Analysis (system comparisons, QA, etc.)
  - Task-based detectability Analysis - Lung Nodules, etc.
  - Selection of reconstruction parameters
    - Control of noise-resolution trade-off

- Measuring noise properties
  - Empirical studies
    - Brute force reconstructions
  - Predictors
    - Closed-form expressions or routines for predicting properties based on sample objects or data
**Tomosynthesis**

- **Limited-angle tomography**
  - Provides limited “depth” resolution
  - Coronal images are often preferred for diagnostics
  - Axial images illustrate the wide blurs (AP direction)

- **Statistical Reconstruction**
  - Better handling of limited-angle data
  - Accounts for non-stationary noise model
  - Typically more difficult to analyze than analytic approaches

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Penalized-Likelihood Properties

- Penalized-Likelihood Reconstruction
  - Forward Model:
    \[
    \vec{y}(\mu) = I_0 \exp(-A\mu)
    \]
    Discretized Object Volume
    Projected Operator
    Number of photons
    Measurements
  - Implicitly defined estimator:
    \[
    \hat{\mu} = \arg \max \Phi(\mu; y) = \arg \max \left[ \log L(\mu; y) - \beta R(\mu) \right]
    \]
    Objective Function
    Log-Likelihood
    Regularization Term
    Poisson Log-Likelihood
    \[
    \log L(y; \mu) = \sum_{i=1}^{N} -y_i([A\mu]_i) - [I_0 \exp(-A\mu)]_i
    \]
  - Noise and resolution properties are
    - Technique- and Object-dependent (nonstationary noise)
    - Shift-variant (Limited-angle geometry, nonstationary noise, penalized-likelihood regularization)

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Predictors for Implicitly Defined Estimators

- Derived by Fessler, IEEE-TMI 1996
  - General Case:
    \[ \text{Cov}\{\hat{\mu}\} \approx [\nabla^2 \Phi(\mu, y)]^{-1} [\nabla^{11} \Phi(\mu, y)] \text{Cov}\{y\} [\nabla^{11} \Phi(\mu, y)]^T [\nabla^2 \Phi(\mu, y)]^{-1} \]
  - For transmission tomography, quadratic penalty:
    \[
    \begin{align*}
    -\nabla^2 \Phi(\mu, y) &= F(\hat{\mu}) + R \\
    \nabla^{11} \Phi(\mu, y) &= -A^T \\
    
    F(\hat{\mu}) &= A^T D[\tilde{y}(\hat{\mu})] A \\
    \text{Backprojection Operator} \\
    \text{(Diagonal) Weighting by modeled variance} \\
    \text{Projection Operator} \\
    \text{Measurement Covariance} \\
    \text{Regularization Operator} \\
    \text{Weighted Projection-Backprojection Operator} \\
    \end{align*}
    \]
    \[
    [\text{Cov}\{\hat{\mu}\}]_j \approx [F(\hat{\mu}) + R]^{-1} \quad A^T \text{Cov}\{y\} A \quad [F(\hat{\mu}) + R]^{-1} e_j
    \]
  - This approximation
    - is closed form
    - relies only on partial derivatives of the objective function
    - depends on the object only via projections of the object
    - is still difficult to evaluate due to matrix inverses

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Fast Computation of Covariance Predictions

\[
[Cov(\hat{\mu})]_j \approx [F(\hat{\mu}) + R]^{-1} A^T \text{Cov}(y) A [F(\hat{\mu}) + R]^{-1} e_j
\]
\[
F(\hat{\mu}) = A^T D[y(\hat{\mu})] A
\]

- Use Fourier Approximations
  - Exploit local shift-invariance
    (Circulant matrix approximation)

\[
[Cov(\hat{\mu})]_j \approx FT^{-1} \left\{ \frac{FT\{A^T \text{Cov}(y) A e_j\}}{\left| FT\{A^T D[y(\hat{\mu})] A e_j + R e_j\}\right|^2} \right\}
\]

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Simulation Data

- **Geometry**
  - Source-Axis: 133 cm
  - Source-Detector: 158 cm
  - 21 Angles: -10° to 10°

- **Detector**
  - 0.5 mm pixels

- **Acquisition**
  - 500 Scans
  - Poisson Noise, 1e4 counts

- **Reconstruction**
  - Quadratic Penalized-Likelihood
  - 0.6 mm voxels, 1000^2 grid
  - 500 iterations SPS algorithm

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Preliminary Real Data Acquisitions

- **Geometry**
  - Source-Axis: 122 cm
  - Source-Detector: 148 cm
  - 41 Angles: -20° to 20°

- **Detector**
  - Varian PaxScan 4030CB
  - 0.388 mm pixels

- **Acquisition**
  - 200 Scans, Point and Shoot
  - 98 KVp, 125 mA, 6.3 ms/shot

- **Reconstruction**
  - Quadratic Penalized-Likelihood
  - 0.4 mm voxels, 1000² grid
  - Poisson Model – 3.7e4 counts
  - 500 iterations SPS algorithm

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Real Data Prediction

- **Caveats for preliminary work**
  - Data covariance is modeled using sample variance
  - Penalized-likelihood reconstruction uses a Poisson model
  - Data is scaled to equivalent number of photons to match mean variance

- **Full cascaded forward model not yet integrated**
  - Predicted data covariance based on x-ray technique, detector physics, etc.
  - Similarly, a more complete model can be used for reconstruction (non-Poisson data, etc.)
Prediction Results
Real Data

Covariance Prediction

Measured Covariance

Sample Location

NPS Prediction

Measured NPS

Horizontal Covariance Profile

Horizontal NPS Profile
Summary/Future Work

- Demonstrated prediction capability
  - Good agreement in simulation studies
  - Preliminary investigations on real data acquisitions

- Future Work
  - Refine real data validation
  - Three-dimensional reconstructions
  - Incorporate into a more general cascaded forward model
    - For input covariance
    - As part of the analysis of an entire imaging chain
  - Incorporate more sophisticated reconstruction methods