Incorporation of Task in 3D Imaging Performance Evaluation: 
The Impact of Asymmetric NPS on Detectability

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ABSTRACT
While analysis of the image noise-power spectrum (NPS) and noise-equivalent quanta (NEQ) are important aspects of imaging system characterization, such metrics are in themselves insufficient descriptions of imaging performance in that they make no account of the imaging task. This paper seeks to quantitatively incorporate imaging task in imaging performance evaluation by combining NEQ with a variety of idealized spatial-frequency-dependent task functions to yield the model observer detectability index. The approach is applied to the case of fully 3D volumetric imaging by flat-panel detector cone-beam CT through analysis of 3D detectability for conditions of varying dose, reconstruction filter, and voxel size. Generalization of the NEQ through incorporation of background “anatomical” noise suggests significant degradation in model observer performance, and the effect is quantified for a variety of detection and discrimination tasks. For anatomical noise modeled according to $1/f^\alpha$ statistics in power-spectral density, the effect is shown to be most severe for low-frequency detection tasks, and somewhat less for mid-to-high spatial frequency tasks, such as discrimination and localization. By considering the fully 3D NEQ, which is known to be asymmetric for cone-beam CT, a compelling hypothesis is realized regarding the detection of structures in volumetric CT images – specifically, that the detectability is different in axial versus sagittal/coronal domains due to asymmetry in the NEQ between these domains, compared to the case in which 3D data are fully interrogated (e.g., by a machine algorithm). This has significant implications for 3-D imaging modalities, including flat-panel cone-beam CT, where the NPS exhibits asymmetric frequency characteristics [viz., high-pass (filtered-ramp) in the transverse domain and low-pass in the longitudinal]. The impact of asymmetric NPS characteristics on detectability was investigated by analysis of the 3D detectability for imaging tasks corresponding to detection and discrimination of fine and low-contrast structures.

Keywords: noise-equivalent quanta, detective quantum efficiency, noise-power spectrum, imaging task, computed tomography, anatomical noise, 3-D imaging, cone-beam CT, flat-panel imagers, imaging performance

1. INTRODUCTION AND MOTIVATION
The development and optimization of novel imaging technology requires an understanding of the physical mechanisms underlying system performance in relation to imaging task. Analysis of spatial-frequency-dependent signal and noise transfer characteristics has proven a powerful, fundamental component of imaging technology innovation – e.g., in the development of flat-panel imagers (FPIs) for radiography, fluoroscopy, and advanced applications. Such analysis includes measurement and modeling of spatial resolution (modulation transfer function, MTF), image noise (NPS), and signal-to-noise ratio (NEQ and detective quantum efficiency, DQE). While these are essential measures of detector performance, they are in themselves insufficient to describe performance in relation to a given task, since they make account of neither the structures of interest in the image nor the response of the (human or machine) observer. Such can be incorporated by integration of NEQ with spatial-frequency-dependent task functions to yield the model observer signal-to-noise ratio, or detectability index. This approach has proven useful in investigation of system optimization and is employed below to examine the impact of asymmetric 3D NPS on a variety of detection and discrimination tasks.

This work is motivated in both a general and specific sense from task-based analysis of system performance. In a general sense, the deployment of advanced imaging technologies should proceed according to quantitative consideration of the task for which the system will be used. As noted by Barrett, "a medical image is always produced for some specific purpose or task, and the only meaningful measure of its quality is how well it fulfills that purpose." In the application of FPIs to cone-beam CT, we have adopted a task-based approach that extends NEQ and related Fourier-based metrics to fully 3D imaging and combines such metrics with 3D descriptions of imaging task.
Figure 1. (a) Experimental benchtop for investigation of advanced flat-panel imaging modalities, including dual-energy imaging, tomosynthesis, and cone-beam CT. Components include: 1.) x-ray tube (Varian Rad94); 2.) flat-panel detector (PerkinElmer RID-1640A); 3.) rotation stage (Compumotor Dynaserv); 4.) translation stages (Parker-Daedal 406XR); 5.) optical bench; and 6.) robust framework for adjustment of imaging geometry and integration with auxiliary tools. (b) Pre-clinical platform for cone-beam CT guidance of surgical procedures, based on an isocentric C-arm (Siemens PowerMobil) and a flat-panel detector (Varian 4030-CB).

Specific motivation for this work is gained from previous studies\(^7,8\) that demonstrate the highly asymmetric nature of the 3D NPS and NEQ for FPI-based cone-beam CT, considered in light of task-based investigation by Myers et al.\(^9\) that demonstrated the significant effect of noise correlation on observer performance. Specifically, it was shown that for systems with equivalent pixel SNR, observer performance was strongly degraded in the presence of higher frequency noise, and that observer performance is confounded by noise correlation that “masquerades” as signal. This is an important consideration for 3D imaging (including helical and cone-beam CT) in which a given volumetric image may present equivalent spatial resolution and pixel SNR throughout, but the detectability of structures varies depending on the domain of visualization due to differences in NPS.

2. ADVANCED IMAGING TECHNOLOGIES FOR INTERVENTIONAL PROCEDURES

Among the advanced applications of FPIs are dual-energy imaging, tomosynthesis, and cone-beam CT. The experimental benchtop in Fig. 1(a) was developed to investigate imaging performance in each of these modalities as part of a program of research targeted at the development and deployment of advanced technologies for image-guided procedures. Such a system provides an experimental proving ground for FPI-based advanced applications. Techniques developed on the experimental benchtop translate directly to pre-clinical platforms, such as the isocentric C-arm shown in Fig. 1(b), under development for image-guided surgery and interventional radiology. An example image from the experimental benchtop is shown in Fig. 2, illustrating the fully 3D nature of FPI-based cone-beam CT image data and the potential for variation in detectability of a given structure between axial and sagittal / coronal domains.

The development of these advanced modalities and their application in image-guided procedures present both challenges and opportunities for a quantitative, task-based approach. Extension of Fourier-based performance metrics to these modalities is underway, providing the essential experimental and theoretical framework for understanding the physical mechanisms that govern imaging performance. We anticipate increased opportunity for task-based analysis across this spectrum of modalities due to reduction in overlying “anatomical” noise (below) and increased applicability of idealized task functions for multi-dimensional image data. For image-guided procedures, the need for image data acquired in a manner well suited to the imaging task is clear, given clinical time constraints and the need for rapid feedback in the interventional procedure. Furthermore, guided procedures offer an opportunity for task-based analysis in that the task can often be specified (i.e., quantified) based on a wealth of diagnostic data, and the imaging system operated in a manner that is tuned optimally to performance of a given task. Finally, given the large data sets associated with multi-dimensional imaging and the time constraints typical of guided procedures, we anticipate an increased role for machine (algorithmic) observers and an increased applicability of simple observer models in task-based analysis.
3. TASK-DRIVEN APPROACH TO IMAGING PERFORMANCE

The task-based approach employed below combines 3D Fourier-based metrics of detector performance – viz., NEQ and DQE – with a variety of idealized 3D task functions to yield the model observer detectability index. In the sections below, the 3D generalized DQE is derived based on previously described cascaded systems analysis of the 3D NPS, combined with a simple model for background “anatomical” noise, and integrated with a variety of idealized task functions in analysis of detectability index. The results are qualitatively compared to a simple human observer study regarding the detectability of objects visualized under varying degrees of noise correlation and asymmetry.

3.1 Noise-Power Spectrum (NPS) Propagation and Asymmetry

As shown previously, the 3D image NPS may be derived from extension of cascaded systems analysis through the processes of image reconstruction. In the 2D projection domain, the NPS, $S_{\text{proj}}(u,v)$, is essentially symmetric by virtue of the transfer characteristics (e.g., scintillator blur and pixel apertures) and is described well by cascaded systems analysis. This symmetry is broken, however, upon application of filters typical of filtered backprojection:

$$S_{\text{proj}}(u,v) = S_{\text{proj}}(u,v) T_{\text{ramp}}^2(u) T_{\text{win}}^2(u) T_{\text{interp}}^2(u,v)$$

(1)

where the ramp filter is applied in the lateral ($u$) domain only. Other filters, such as apodization, $T_{\text{win}}$, and interpolation, $T_{\text{interp}}$, may be applied symmetrically or asymmetrically. Analogous to the case of transaxial CT as described by Kijewski et al., the 3D NPS is formed by superposition of vanes of the 2D filtered NPS in the 3D Fourier domain:

$$S_{\text{recon}}(f, f_z) = \frac{\pi}{f} S_{\text{proj}}(f, f_z) \text{III}(f, f_z)$$

(2)

where $f$ and $f_z$ are radial and longitudinal spatial-frequency coordinates for the 3D Fourier domain. Such analysis has shown reasonable agreement with the measured 3D NPS of flat-panel cone-beam CT based on indirect-detection FPIs and suggests asymmetry in the NPS dependent on the detector type (e.g., direct or indirect) and choice of applied filters.

3.2 Generalized DQE

The 3D DQE and NEQ for FPI-based cone-beam CT has been derived previously. In this paper, we incorporate the effects of background “anatomical” noise as an additional noise term to yield the “generalized” DQE in the presence of background fluctuations. The background NPS, $S_{\beta}(f)$, is modeled as “one-over-$f^\beta$” noise proportional to $K_\beta/f^\beta$ with $\beta$ in the range 2 to 4 for radiography, and varied from 0 to 4 for 3D imaging. The generalized DQE is therefore:

$$DQE_{\text{gen}}(f_x, f_y, f_z) = S_{\text{deterministic}}(f_x, f_y, f_z) S_{\text{recon}}(f_x, f_y, f_z) + S_B(f_x, f_y, f_z)$$

(3)

where $S_{\text{deterministic}}$ is the ‘blur-only’ NPS associated with transfer of quantum noise through the imaging system, $S_{\text{recon}}$ is the NPS of the actual imaging system, and $S_B$ is the NPS associated with background structures. Example results are shown in Figs. 3 and 4, illustrating significant asymmetry in the 3D DQE and the degradation due to “anatomical” noise.
Figure 3. (a) and (b) Axial and sagittal NPS. (c) and (d) Axial and sagittal DQE. (e) and (f) Axial and sagittal generalized DQE, respectively. Correlation and asymmetry in the system noise transfer characteristics are evident in the 3D NPS (a) and (b). The significant impact of “anatomical” noise on performance is evident in comparison of DQE (c) and (d) with the generalized DQE (e) and (f) for the case $E = 2.0$. Note the significant reduction in DQE due to “anatomical” noise at low spatial frequencies.

Figure 4. (a) and (b) DQE in axial and sagittal domains for various choices of reconstruction filter. The differences are attributable to 3D aliasing. (c) and (d) Generalized DQE for various values of $\beta$ (“clutter”). (e) and (f) Generalized DQE for direct and indirect-detection FPIs differing only in presampling MTF (i.e., equivalent quantum detection efficiency, gain, additive noise, etc.). Inclusion of anatomical noise dramatically affects task-based optimization by shifting emphasis to mid and high frequencies.
Figure 5. Example hypothesis-testing task functions. In each case, the task function is computed from the difference of Fourier transforms between two hypotheses, A and B – e.g., signal present and signal absent, respectively. The curves shown illustrate four example tasks: 1.) Delta-function detection task (A: delta-function present; B: uniform background), which weights all spatial frequencies uniformly; 2.) Gaussian detection task (A: gaussian sphere present of diameter \( d_{\text{task}} \); B: uniform background), which weights low spatial-frequencies; 3.) Discrimination of a gaussian sphere from a noisy background (A: gaussian sphere present; B: noisy background), which weights high spatial frequencies; and 4.) Localization of a gaussian sphere (A: gaussian sphere at position 1; B: gaussian sphere at position 2), which weights mid-frequencies, depending on the size and separation of the object functions.

3.3 Task Functions and Model Observer Detectability Index

Imaging task is combined with Fourier-based NEQ descriptions through generation of spatial-frequency-dependent task functions as described in ICRU Report 54. Given two hypotheses (e.g., A: signal present; B: signal absent) the idealized hypothesis-testing task function is derived from the Fourier difference:

\[
T_{\text{task}}(f) = |FT[H_A(\vec{x})] - FT[H_B(\vec{x})]|^2
\]  

Simple examples used previously\(^4\,^5\) to investigate optimization of FPI-based cone-beam CT systems are illustrated in Fig. 5, including: delta-function detection against uniform background; gaussian detection against uniform background; gaussian discrimination from noisy background, localization of gaussian spheres; and size estimation of gaussian spheres.\(^14\,^16\) These idealized task functions essentially operate as weighting functions on the generalized NEQ, identifying the spatial frequencies of importance in performing a given task. Combining the two yields the ideal observer signal-to-noise ratio, or detectability index, adapted here to the 3D Fourier domain:

\[
d' = \int \int \int_{\text{Nyq}} \text{NEQ}_{\text{gen}}(\vec{f}) T_{\text{task}}(\vec{f})^2 d\vec{f}
\]  

The detectability index provides a practical Fourier-based figure of merit for imaging performance, which can be directly related to observer performance in receiver operating characteristic (ROC) tests. Still, it is important to recognize the constraints and limitations of a Fourier-based approach. First among these are assumptions of linearity, stationarity, and shift-invariance – properties strictly obeyed by no physical imaging system. The former two may be demonstrated “locally,” – i.e., over finite extent in space, time, signal size, etc. – and the latter has been addressed through modified assumptions of cyclo-stationarity\(^10\) as well as the construction of “expectation” values that average a given Fourier metric over all possible locations of the input.\(^17\) Furthermore, variability in the location of the input signal and subsequent averaging over all locations effectively minimizes off-diagonal elements of the crosstalk matrix and imparts “a sort of stationarity” on the system.\(^18\) Thus, variability in the input location reduces the crosstalk matrix to a form similar to the NEQ and renders Fourier-based metrics applicable over a broad range of realistic conditions.

As shown in Fig. 6, detectability was computed for a variety of imaging tasks as a function of object size, background “anatomical” noise, and 3D reconstruction parameters for FPI-based cone-beam CT system shown in Fig. 1(a). Figure 6(a) shows the significant reduction in detectability with increasing anatomical “clutter,” \( \beta \), (constant total background noise-power). Performance decreases as a function of object size (constant total signal power), in the presence of background noise, suggesting a reduction in detectability as the anatomical noise “masquerades” as the structure of interest. [In the absence of background noise, detectability increases with object size as shown in Fig. 6(e).] The detectability index computed for various idealized task functions is shown in Figs. 6(b-f). As evident in Fig. 6(b), detectability suffers to a greater extent in the presence of background clutter for the low-frequency detection task than for the higher-frequency localization task. As shown in Figs. (c-d), incorporation of anatomical noise completely changes the dependencies for studies of optimization – e.g., optimal apodization filter, parameterized by \( h_{\text{w}\beta} \) ranging from 0.5 (Hanning) to 1.0 (Ram-Lak). Similarly in Fig. 6(e-f), generalization of NEQ has a dramatic effect on the detectability index predicted for low and mid-frequency tasks.
Figure 6. Detectability index as a function of background “anatomical” noise, object size, and 3D reconstruction parameters in axial (solid) and sagittal (dashed) domains. (a) Detectability index vs. object size and background for a gaussian detection task (1 mm gaussian sphere). The reduction in $d'$ with increasing object size is purely due to the influence of background noise. (b) Detectability vs. anatomical “clutter” for gaussian detection and localization tasks. Note the steep falloff in the lower-frequency task due to background clutter. (c and d) Detectability versus apodization filter (characterized by the parameter $h_{\text{win}}$, 0.5 for Hanning and 1.0 for sharp Ram-Lak). In the absence of background noise (c), detectability is a weak function of apodization, typically reducing due to increased noise aliasing with sharper filters. In the presence of background noise (d), however, detectability improves for sharper filters. (e and f) Detectability versus object size. In the presence of background noise, detectability is dramatically reduced for the low-frequency detection task under conditions where the noise “masquerades” as signal. The effect of NPS correlation and asymmetry in axial versus sagittal domains is seen to depend on a fairly complex interplay between the generalized NEQ and task function.

3.4 Measurement of Human Observer Detection Threshold

A simple human observer study was performed to determine detectability thresholds (i.e., minimum detectable object size) in axial and sagittal planes under conditions of varying correlation and asymmetry in axial and sagittal views. A phantom consisting of seven acrylic spheres suspended in gelatin was constructed to give relative contrast close to the detectability limit. Spheres ranged in diameter from 1.6 to 12.7 mm. Cone-beam CT images were acquired on a previously described experimental bench at dose levels spanning the sensitive range of the detector (CsI-based Varian 4030CB). Images were reconstructed at voxel size ($a_{\text{vx}}$ in the axial domain; $a_z$ in the longitudinal direction) ranging from 0.25 to 2.0 mm and using apodization filters ranging from $h_{\text{win}} = 0.5$ (Hanning) to 1.0 (Ram-Lak). Axial and sagittal views of each sphere formed the basis of image data presented to observers. In these preliminary studies, ten non-expert observers (physicists and engineers) were shown a series of seven images (each containing a sphere of different diameter) and were asked to identify the smallest sphere that could be confidently detected. Axial and sagittal images were viewed separately, with randomized reading order following a period of reader training. Observers were free to adjust their viewing distance and perspective, but magnification and window/level were fixed.

Example axial and sagittal images are shown in Figs. 7 and 8 at various voxel size ($a_{\text{vx}}$, $a_z$) and reconstruction filter ($h_{\text{win}}$). Images of the largest (12.7 mm) sphere are shown in Fig. 7 at various axial and longitudinal voxel size – $a_{\text{vx}}$ and $a_z$, respectively – where the two are equal only along the diagonal in each case. The strong dependence of detectability on $a_{\text{vx}}$ is simply due to improved contrast-to-noise ratio, given the strong (inverse cube-root) dependence of voxel noise on voxel size. Off-diagonal images in the sagittal domain represent cases of strong asymmetry.
Figure 7. Example images of the 12.7 mm acrylic sphere suspended in gelatin, viewed in axial and sagittal domains for various axial and longitudinal voxel size – $a_x$ and $a_z$, respectively. For the sagittal images, examples along the diagonal have symmetric voxel size, but asymmetric NPS characteristics, while images off-diagonal exhibit strong asymmetry in spatial resolution and noise.

Figure 8. Example images of acrylic spheres in gelatin reconstructed using various apodization filters ($h_{win}$). In addition to the reduction in contrast-to-noise ratio expected for sharper apodization filters, there is a marked influence of asymmetry upon detectability – e.g., in a given column of sagittal images compared to the respective column of axial images.
Results of the detection threshold measurements are summarized in Fig. 9, in which the minimum detectable sphere diameter (descending order such that smaller spheres are higher on the axis) is plotted versus longitudinal voxel size (“slice thickness,” \( a_z \)). In each case, results show the minimum detectable sphere size for axial (solid lines) and sagittal (dashed lines) domains, and the increase in detection threshold with slice thickness is in large part a result of reduced voxel noise for larger slice thickness. In the first group (\( a_{xy} = 0.25 \) mm), however, the detection threshold is markedly different between axial and sagittal domains, although voxel noise is the same in each case, suggesting significant degradation in detectability due to highly asymmetric sagittal noise-power characteristics. This discrepancy is reduced in the second group (\( a_{xy} = 2.0 \) mm), where for large \( a_z \) (upper right corner; symmetric voxels) the detection threshold is approximately the same between axial and sagittal domains, and the trend is slightly reversed at small \( a_z \), where detection was slightly improved in the sagittal case. These results suggest a complex interplay between noise correlation and asymmetry that is consistent with trends revealed in calculation of detectability index.

4. DISCUSSION AND CONCLUSIONS

The development of advanced imaging technologies, such as FPI-based cone-beam CT requires quantitation of imaging performance based on meaningful figures of merit. While Fourier metrics such as MTF, NPS, and NEQ are valuable descriptors of detector performance, they are insufficient as metrics of overall performance in that they make no account of the imaging task. Incorporation of spatial-frequency-dependent task functions with the NEQ provides a practical, task-driven approach to performance evaluation for a wide variety of imaging conditions and tasks. Generalization of the NEQ to include the effects of background “anatomical” noise is an important component of such analysis, since such can far outweigh the influence of quantum and electronics noise traditionally accounted in the NEQ. The general task-based approach provides a quantitative framework that integrates linear systems analysis of detector performance (including transfer characteristics for various detector types, 3D reconstruction, and noise aliasing effects) with a simple model for anatomical background (modeled as one-over-\( f \) noise-power) and a variety of idealized task functions.

Asymmetrically correlated noise-power was shown to have a significant influence on the detectability index and the performance of human observers, in general agreement with the results of Myers et al. in that performance was found to degrade under conditions of strong noise correlation and asymmetry. Background “anatomical” noise severely reduced detectability index for low-frequency detection tasks, particularly in cases where the background noise and object of interest had similar spatial frequency characteristics. The detectability index was investigated in axial and sagittal domains, revealing a complex interplay between detector type, 3D reconstruction filter, imaging task, and background noise. The development of advanced imaging technologies for image-guided procedures offers a challenging and promising application of such a task-based approach, requiring extension of generalized NEQ to advanced modalities and identification of idealized task functions appropriate to specific guided procedures. A practical, task-based approach is well suited to such development in that tasks can often be specified (i.e., quantified) based on a wealth of diagnostic image data. Furthermore, given the size of image data and the need for rapid feedback in guided procedures, we anticipate increased use of machine algorithmic observers in such procedures, which may be more simply described than a human observer using simple, linear observer models and representations of imaging task.
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