Cone-beam computed tomography with a flat-panel imager: Magnitude and effects of x-ray scatter

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A system for cone-beam computed tomography (CBCT) based on a flat-panel imager (FPI) is used to examine the magnitude and effects of x-ray scatter in FPI-CBCT volume reconstructions. The system is being developed for application in image-guided therapies and has previously demonstrated spatial resolution and soft-tissue visibility comparable or superior to a conventional CT scanner under conditions of low x-ray scatter. For larger objects consistent with imaging of human anatomy (e.g., the pelvis) and for increased cone angle (i.e., larger volumetric reconstructions), however, the effects of x-ray scatter become significant. The magnitude of x-ray scatter with which the FPI-CBCT system must contend is quantified in terms of the scatter-to-primary energy fluence ratio (SPR) and scatter intensity profiles in the detector plane, each measured as a function of object size and cone angle. For large objects and cone angles (e.g., a pelvis imaged with a cone angle of 6°), SPR in excess of 100% is observed. Associated with such levels of x-ray scatter are cup and streak artifacts as well as reduced accuracy in reconstruction values, quantified herein across a range of SPR consistent with the clinical setting. The effect of x-ray scatter on the contrast, noise, and contrast-to-noise ratio (CNR) in FPI-CBCT reconstructions was measured as a function of SPR and compared to predictions of a simple analytical model. The results quantify the degree to which elevated SPR degrades the CNR. For example, FPI-CBCT images of a breast-equivalent insert in water were degraded in CNR by nearly a factor of 2 for SPR ranging from ~2% to 120%. The analytical model for CNR provides a quantitative understanding of the relationship between CNR, dose, and spatial resolution and allows knowledgeable selection of the acquisition and reconstruction parameters that, for a given SPR, are required to restore the CNR to values achieved under conditions of low x-ray scatter. For example, for SPR=100%, the CNR in FPI-CBCT images can be fully restored by: (1) increasing the dose by a factor of 4 (at full spatial resolution); (2) increasing dose and slice thickness by a factor of 2; or (3) increasing slice thickness by a factor of 4 (with no increase in dose). Other reconstruction parameters, such as transaxial resolution length and reconstruction filter, can be similarly adjusted to achieve CNR equal to that obtained in the scatter-free case. © 2001 American Association of Physicists in Medicine. [DOI: 10.1118/1.1339879]

Key words: flat-panel imager, cone-beam computed tomography, x-ray scatter, scatter-to-primary ratio, artifacts, contrast, noise, contrast-to-noise ratio

I. INTRODUCTION

The combination of active matrix flat-panel imagers (FPIs) and cone-beam computed tomography (CBCT) represents a promising technology for volumetric imaging, 1–8 capitalizing on continuing advances in FPI technology and cone-beam reconstruction techniques. FPIs provide efficient, distortionless, real-time detectors that are experiencing widespread proliferation in x-ray projection imaging, and cone-beam reconstruction 9–12 techniques have been accelerated from hours to seconds through the development of dedicated hardware. Capable of acquiring volumetric (i.e., multi-slice) images in an open geometry, from a single rotation about the patient, and without the need for a ring-based gantry or a mechanism for translating the patient, FPI-CBCT provides separation of the imaging system and patient support. These aspects appear particularly advantageous for image-guided therapies, such as radiation therapy, brachytherapy, and surgery, where the logistics of the treatment procedure largely dictate the geometry. Furthermore, such systems could provide the therapist with combined radiographic, fluoroscopic, and tomographic imaging. As described previously, 1,13 an FPI-CBCT system is being developed for online tomographic guidance of radiation therapy procedures by incorporating a kilovoltage x-ray tube and an FPI on the gantry of a medical linear accelerator. This system is expected to provide soft-tissue visibility and spatial resolution sufficient for correction of patient setup errors and interfraction organ motion. 1,13

The FPI-CBCT system under development has demonstrated reasonable volumetric uniformity, noise, and spatial resolution characteristics, providing soft-tissue visibility comparable to that achieved with a conventional CT scanner. 1 Application of this technology, however, is not without its challenging aspects, including the effects of detector performance, image lag, and x-ray scatter. Analysis of FPI signal and noise performance 1,5 suggests that in order to provide high-quality projections at very low exposures (e.g., on the order of a μR to the detector for lateral projections of
a large pelvis), high-performance FPIs are required (e.g., incorporating a CsI:Tl x-ray converter and low-noise amplifiers). Analysis of the magnitude and effects of image lag in FPI-CBCT suggests that image lag can result in subtle artifacts in regions of high-contrast objects at high exposures, and that such effects can be largely eliminated through simple procedural and/or algorithmic methods. This article is concerned with the magnitude and effects of x-ray scatter in flat-panel cone-beam CT and seeks to identify strategies for management of deleterious scatter effects.

Investigations of x-ray scatter in fan-beam CT have demonstrated experimentally and analytically that scatter results in artifacts (e.g., cup and streak artifacts) and quantitative inaccuracy in reconstructed CT number (CT#).14–16 Analytical and Monte Carlo methods have been employed to estimate the intensity of scattered radiation at diagnostic energies,17–21 demonstrating reasonable agreement with measured results. Based on the magnitude and shape of such intensity distributions, correction algorithms22,23 have been developed that largely remove scatter artifacts and restore CT# accuracy. Such methods are standard components of modern fan-beam CT systems. In cone-beam CT, the problem of x-ray scatter is expected to be significantly greater, due to the use of a large cone angle and 2D detector. A significant degree of scatter rejection can be achieved using conventional grids and focused collimators;24 however, it is uncertain whether such strategies provide an advantage compared to a knowledgeably selected air gap.6,25 For digital imagers, Netizel25 concluded that air gaps and grids perform comparably under high-scatter conditions, and for low-scatter conditions an air gap is clearly superior; moreover, the advantages of a gap are even stronger if the application, such as FPI-CBCT, can logistically accommodate large air gaps.

This article reports on the magnitude of x-ray scatter expected in the clinical environment and quantifies the effects on image artifacts, CT# inaccuracy, contrast, noise, and contrast-to-noise ratio (CNR) in FPI-CBCT reconstructions. Finally, the degree to which CNR can be restored to levels achieved under conditions of low x-ray scatter is examined using simple analytical forms for CT contrast14 and noise.26

II. METHODS AND MATERIALS

A. Experimental setup

The magnitude and effects of x-ray scatter in flat-panel cone-beam CT were measured using the experimental setup illustrated in Fig. 1. The imaging geometry [i.e., source-to-axis distance (y\textsubscript{AOR}), source-to-detector distance (y\textsubscript{FPI}), and axis-to-detector distance (y\textsubscript{ADD})] mimics that of a system for online tomographic guidance of radiotherapy procedures1,13 and, for conditions of high x-ray scatter, is close to the optimal configuration,6 considering the effects of geometric sharpness, imaging task, scatter rejection, and detector performance. As detailed elsewhere,1,3 the three main components of the system are an x-ray tube, a rotating object, and an FPI. For rectangular collimation and field of view, the angle subtended by the primary x-ray beam in the lateral (x)

direction is the fan angle, \( \phi\text{fan} \), and determines the lateral field of view, FOV\textsubscript{FPI} and FOV\textsubscript{AOR} at the axis and detector planes, respectively:

\[
\phi\text{fan} = 2 \tan^{-1} \left( \frac{\text{FOV}^\text{AOR}}{2y\text{AOR}} \right) = 2 \tan^{-1} \left( \frac{\text{FOV}^\text{FPI}}{2y\text{FPI}} \right),
\]

Similarly, the angle subtended by the beam in the longitudinal (z) direction is the cone angle, \( \phi\text{cone} \):

\[
\phi\text{cone} = 2 \tan^{-1} \left( \frac{\text{FOV}^\text{AOR}}{2y\text{AOR}} \right) = 2 \tan^{-1} \left( \frac{\text{FOV}^\text{FPI}}{2y\text{FPI}} \right).
\]

where FOV\textsubscript{AOR} and FOV\textsubscript{FPI} are the z-extent of the primary radiation beam at the axis of rotation (AOR) and at the FPI, respectively. The x-ray tube was a General Electric (Milwaukee, WI) Maxiary 75 operated at 120 kVp (measured) with 0.5-mm Cu added filtration. Exposure was measured using an RTI Electronics (Molndal, Sweden) PMX-III multimeter and is reported either in terms of the exposure in air at iso-center, \( X\text{iso} \), or at the center of the FPI, \( X\text{FPI} \). A typical FPI-CBCT acquisition involved 300 projections at 1 mAs per projection (at which the FPI exhibits linear response and is strongly input-quantum-limited), giving a total scan exposure in air of \( X\text{iso} \sim 4.6 \times 10^{-8} \) C/kg (\( \sim 1.8 \) R). The exposure per projection at the center of the FPI ranged from \( X\text{FPI} \sim (1.3–7.7) \times 10^{-8} \) C/kg (i.e., \( \sim 50–300 \) \( \mu \)R, corresponding to \( \sim 0.5\%–3\% \) of sensor saturation,\textsuperscript{3} depending on object thickness).

A simple phantom was constructed to allow volumetric imaging under conditions of variable x-ray scatter. The essential requirements in the phantom design included: (1) al-

![Figure 1. Schematic illustration of the experimental setup used to measure the magnitude and effects of x-ray scatter in flat-panel cone-beam CT. The x-ray tube, rotating object, and FPI operate synchronously to acquire projection data for cone-beam reconstruction. The magnitude of x-ray scatter was varied through adjustment of the fan angle ($\phi_{fan}$), the cone angle ($\phi_{cone}$), and the thickness of PMMA ($T_{PMMA}$) surrounding a rotating water-filled cylinder. The Pb blocker was used for measurement of the SPR at the detector plane.](image)
lowance of FPI-CBCT imaging without lateral truncation of the projection data; (2) composition approximating water; and (3) variation in phantom thickness across a range similar to that of human anatomy. As illustrated in Fig. 1, the phantom consisted of a water-filled cylinder (11 cm diameter) surrounded by slabs of polymethyl methacrylate (PMMA). Note that only the cylinder rotated during FPI-CBCT acquisition, and the PMMA slabs (supported on a platform above the rotation stage) were stationary and did not impinge on the volume of reconstruction. Thus reconstructed images show only the water-filled cylinder and not the surrounding PMMA. The magnitude of x-ray scatter at the detector plane was varied by adjusting the fan angle (two settings, \( \phi_{\text{fan}} = 14^\circ \) and 22\(^\circ\); see below), the cone angle (varied continuously, and/or the thickness of PMMA, \( T_{\text{PMMA}} \) (across a range corresponding to the AAPM standard phantoms for measurement of entrance exposure\(^{27}\)). In order to examine the effects of x-ray scatter alone, without the confounding influence of beam-hardening\(^{27}\) that results from variation in \( T_{\text{PMMA}} \), FPI-CBCT reconstructions are shown for cases in which \( \phi_{\text{cone}} \) alone was varied (with \( \phi_{\text{fan}} \) and \( T_{\text{PMMA}} \) fixed).

The FPI was the same as that used in previous studies of FPI-CBCT performance,\(^{1-4} \) manufactured by PerkinElmer Optoelectronics and based on a 512 × 512 matrix of a-Si:H photodiodes and thin-film transistors (TFTs) at 400-\( \mu\)m pixel pitch. The FPI incorporates a 133-mg/cm\(^2\) \( \text{Gd}_2\text{O}_3:\text{ Tb} \) x-ray converting screen and can be addressed at up to five frames per second (fps) at 16-bit precision. FPI-CBCT acquisition involved a repeated, synchronized sequence of: (1) delivery of a radiographic exposure; (2) reading of the FPI projection image; and (3) incremental rotation of the object. For all scans, 300 projections were acquired, with incremental rotation of the object by 1.2\(^\circ\) through 360\(^\circ\) and with the FPI offset from a symmetrical position by 1/4 the detector element spacing.\(^{28} \) Since the FPI was operated at a fairly low frame rate of 0.625 fps, a complete scan took 8 min. Cone-beam reconstructions [512 × 512 × (1–512) voxels] were performed using a modified FDK algorithm\(^9\) for filtered back-projection on a Sun (Fremont, CA) UltraSparc workstation.

The range in selected fan angle, cone angle, and object thickness correspond to conditions expected in the clinical setting. The FPI described above is too small (20.5×20.5 cm\(^2\)) for volumetric imaging of large anatomy in this geometry; therefore, a larger FPI\(^{29} \) (41×41 cm\(^2\) active area, not tiled) forms the basis of the clinical prototype that is being implemented on a medical linear accelerator in either of two geometries: (1) a “centered” geometry (illustrated in Fig. 1); and (2) an “offset” geometry in which the detector is offset from the central axis by up to half the FPI width.\(^{30} \) With the larger FPI, the centered geometry allows imaging of objects with a maximum width of \( \sim 25.5 \) cm (\( \phi_{\text{fan}} \sim 14^\circ \)) without truncation of the projection data—sufficient for imaging of head, neck, and extremity sites. Imaging of larger anatomy (e.g., a pelvis with lateral extent up to \( \sim 40 \) cm) is accomplished using the offset geometry with approximately the same fan angle. Imaging of such large anatomy without truncation in a centered geometry would require a detector width of \( \sim 64 \) cm (\( \phi_{\text{fan}} \sim 22^\circ \)). Measurements were performed using both \( \phi_{\text{fan}} = 14^\circ \) and 22\(^\circ\), with results shown for the former unless otherwise stated. The cone angle was varied from \( \phi_{\text{cone}} \sim 0.5^\circ \) (corresponding to \( \text{FOV}_{\text{z,AOR}} = 0.9 \) cm and \( \text{FOV}_{\text{z}} = 1.4 \) cm, i.e., \( \sim 35 \) slices in a full-resolution reconstruction) to \( \phi_{\text{cone}} \sim 10.5^\circ \) (corresponding to \( \text{FOV}_{\text{z,AOR}} = 19 \) cm and \( \text{FOV}_{\text{z}} = 30 \) cm, i.e., \( \sim 750 \) slices).

### B. Magnitude of x-ray scatter

The SPR at the detector plane was measured in a manner similar to that of Johns and Yaffe.\(^{14} \) As shown in Fig. 1, a Pb blocker (\( \sim 9 \)-mm-diam disk, \( \sim 10 \) mm thick) was placed at the entrance of the phantom on the central axis of the beam. For each measurement of SPR, ten projections were acquired with the FPI, five with the blocker in place and five with the blocker removed. The ensemble of pixel values in the shadow of the blocker from the former five gave the “scatter only” signal, and that in the latter five gave the “scatter + primary” signal. The pixel dark values (i.e., the “offsets”) were subtracted from each image, and tube output fluctuations were corrected by normalizing each image according to the measured exposure. The SPR was obtained by dividing the “scatter only” signal by the difference between the “scatter only” and “scatter+primary” signal. Off-focal radiation\(^{14} \) was assumed negligible, and the energy response of the FPI (i.e., the signal per incident fluence) was assumed equivalent for the primary and scattered x-ray spectra. Measurements of SPR were performed using Pb blockers ranging in diameter from \( \sim 5 \) to 15 mm in order to determine the correction factor associated with nonzero disk size. The factor was determined from a linear fit to the results (SPR versus disk diameter) extrapolated to zero disk size. For example, for a medium-sized phantom (15-cm PMMA) the correction factor was \( \sim 1.017 \) for the nominal 9-mm disk (i.e., SPR values reported were increased by \( \sim 1.7\% \) from the measured values).

Measurements were first performed in order to benchmark the observed SPR to values corresponding approximately to human anatomy. For these measurements, the water-filled cylinder was removed, and the PMMA slabs were placed in simple rectangular arrangements approximating the AAPM standard phantoms\(^{27} \) for exposure measurement of various anatomical sites (e.g., \( \sim 5 \)-cm PMMA for “extremity,” \( \sim 10 \) cm for “chest,” \( \sim 18 \) cm for “abdomen,” and \( \sim 30 \) cm referred to herein as “pelvis”). Only in these measurements was the phantom intended to approximately represent human anatomy. The SPR was also measured for each configuration of \( \phi_{\text{fan}} \), \( \phi_{\text{cone}} \), and \( T_{\text{PMMA}} \) used in measurements (viz., scatter-fluence distributions, artifacts, contrast, and noise; see below) in which PMMA slabs surrounded the water-filled cylinder as in Fig. 1. In those measurements, the PMMA does not necessarily represent human anatomy (e.g., in the case of the water-filled cylinder flanked by sidelobes of PMMA); rather, the PMMA thickness was merely one means of continuously varying the SPR at the detector.

A second type of measurement was performed to determine the spatial distribution of x-ray scatter in the detector plane, again using the geometry of Fig. 1. First, a series of 20
“low scatter” projection images of the cylinder were acquired under conditions that minimized SPR [i.e., small fan angle, cone angle ~0.5°, and \( T_{\text{PMMA}} = 0 \text{ cm} \), for which SPR = \( (2.10 \pm 0.27)\% \)]. Then, without moving the cylinder, projection images were acquired as a function of SPR by increasing \( \phi_{\text{fan}} \), \( \phi_{\text{cone}} \), and \( T_{\text{PMMA}} \). Each series of 20 images was averaged, corrected for offset variations, and normalized to a constant value in the unattenuated beam. The spatial distribution of x-ray scatter energy fluence in the detector plane was computed from the difference between the images acquired as a function of SPR and the image acquired under “low-scatter” conditions. Knowledge of the magnitude and shape of such x-ray scatter distributions is an important component of scatter correction algorithms, which are the subject of on-going investigation.

C. Shading artifacts and CT number inaccuracy

Two types of shading artifact were measured in transaxial slices of FPI-CBCT images: (1) the artifact in which voxel values in the image of a uniform water cylinder are reduced and nonuniform, forming a “cup”; and (2) the artifact in which the voxel values between two dense objects are reduced, forming a “streak.” For the former, images of the water-filled cylinder were acquired as a function of \( \phi_{\text{cone}} \) and \( T_{\text{PMMA}} \). To quantify the inaccuracy of voxel values, the mean value (\( \mu \)) in the water-filled interior of the cylinder was compared to the attenuation coefficient for water (\( \mu_{\text{H}_2\text{O}} = 0.020 \text{ mm}^{-1} \)), computed from the energy-dependent attenuation coefficient and the x-ray spectrum of the primary beam, yielding the inaccuracy:

\[
\Delta = 100 \times \frac{\langle \mu \rangle - \mu_{\text{H}_2\text{O}}}{\mu_{\text{H}_2\text{O}}}.
\]

(2a)

To quantify the degree of spatial nonuniformity, voxel values near the center of the reconstruction, \( \mu_{\text{center}} \), were compared to those at ~5 mm inside the edge of the cylinder, \( \mu_{\text{edge}} \), giving the degree of “cupping”:

\[
t_{\text{cup}} = 100 \times \frac{\mu_{\text{edge}} - \mu_{\text{center}}}{\mu_{\text{edge}}}.
\]

(2b)

The streak artifact was measured using two 2.8-cm-diam “bone” inserts placed within the water cylinder [SB3 cortical bone from the Gammex RMI (Middleton, WI) electron density phantom, with specified electron density, \( \rho_e \), of 1.707 times that of water, physical density, \( \rho \), of 1.84 g/cm³, and approximate CT# of 1367.8]. Although the small scale of the phantom prohibits direct interpretation of the results with respect to large human anatomy (e.g., concerning the cup artifact in a transaxial image of a pelvis, or the streak artifact between femoral heads), it does provide qualitative visualization of the magnitude of such artifacts as a function of SPR across a range that is representative of that anticipated in the clinical setting.

D. Effect of x-ray scatter on contrast

The effect of x-ray scatter on object contrast was investigated using a “breast-equivalent” insert placed within the water cylinder [BR SR1 breast from the Gammex RMI electron density phantom, with specified \( \rho_e \), of 0.980 times that of water, \( \rho \) of 0.99 g/cm³, and approximate CT# of ~46.7]. Contrast is defined as the difference between the ensemble average of voxel values in an insert compared to that in water (adjacent to and at the same radius as the insert) and was measured as a function of SPR. FPI-CBCT images of the contrast phantom were acquired as a function of SPR through variation of \( \phi_{\text{cone}} \) and \( T_{\text{PMMA}} \), and the resulting degradation in contrast was compared to the following analytical description.

Following Johns and Yaffe, we consider the primary and scatter fluence (\( P \) and \( S \), respectively) behind a uniform cylinder of diameter \( d \) and attenuation coefficient \( \mu \). In the unattenuated beam, the primary and scatter fluence are \( P_0 \) and \( S_0 \), respectively, giving for the measured value \( \hat{\mu} \):

\[
\hat{\mu} = \frac{1}{d} \lim \left( \frac{P}{P_{\text{e}^{\hat{S}d}}} + \ln \left( \frac{1 + S_0/P_0}{1 + S/P} \right) \right).
\]

(3a)

Therefore,

\[
\frac{\hat{\mu}_1}{d} + \frac{1}{d} \ln \left( \frac{P_0}{P_{\text{e}^{\hat{S}d}}} + \ln \left( \frac{1 + S_0/P_0}{1 + S/P} \right) \right).
\]

(3b)

Since the second term is negative, scatter causes voxel values in the reconstruction to be lower than the true attenuation coefficient. We now consider a second object of diameter \( d_2 \) and attenuation coefficient \( \mu_2 \) that is contained within the first (e.g., as the breast-equivalent insert is contained in the water cylinder). We take \( \alpha \) to be the relative size of the second object, such that \( d_2 = \alpha d \), and define \( \delta \) to be the true difference in attenuation coefficients. With \( \hat{\mu}_1 \) and \( \hat{\mu}_2 \) representing measured values of \( \mu_1 \) and \( \mu_2 \), respectively, and considering the diameter, \( d \), such that \( d = d_1 + d_2 \) and \( d_1 = (1 - \alpha)d \), we have:

\[
\mu_1 \hat{d}_1 + \mu_2 \hat{d}_2 = \ln \left( \frac{P_0}{P_{\text{e}^{\hat{S}d}}} + \ln \left( \frac{1 + S_0/P_0}{1 + S/P} \right) \right),
\]

\[
\mu_1 (1 - \alpha)d + \mu_2 \alpha d = \ln \left( \frac{P_0}{P_{\text{e}^{\hat{S}d}}} + \ln \left( \frac{1 + S_0/P_0}{1 + S/P} \right) \right),
\]

\[
\alpha (\hat{\mu}_1 - \hat{\mu}_2) = \frac{1}{d} \ln \left( \frac{P_0}{P_{\text{e}^{\hat{S}d}}} + \ln \left( \frac{1 + S_0/P_0}{1 + S/P} \right) \right),
\]

\[
\hat{\mu}_1 - \hat{\mu}_2 = \frac{1}{\alpha d} \ln \left( \frac{P_0}{P_{\text{e}^{\hat{S}d}}} + \ln \left( \frac{1 + S_0/P_0}{1 + S/P} \right) \right),
\]

which is the measured contrast, \( \hat{C} \). Combining Eqs. (3b) and (3c) gives:
\[ \hat{C} = \frac{1}{\alpha_d} \left[ \ln \left( \frac{P_0}{P} \right) + \ln \left( \frac{1+S_0/P_0}{1+S/P} \right) \right] - \frac{1}{\alpha_d} \ln \left( \frac{P_0}{Pe^{\delta d}} \right) 
+ \ln \left( \frac{1+S_0/P_0}{1+S/P} \right) \right], \\
= \frac{1}{\alpha_d} \left[ \ln \left( \frac{P_0}{P} \frac{Pe^{\delta d}}{P_0} \right) + \ln \left( \frac{1+S_0/P_0}{1+S/P} \right) \right], \\
= \delta + \frac{1}{\alpha_d} \ln \left( \frac{1+S/P}{1+S_0/P_0} \right). \tag{3d} \]

The measured contrast, \( \hat{C} \), differs from the actual difference in linear attenuation coefficients, \( \delta \), by a term related to the SPR, and since the absolute magnitude of this term increases with SPR, the result is degradation in contrast. For example, considering \( \mu_2 < \mu_1 \) (i.e., the second object is ‘‘dark’’), then \( \delta \) is positive and the second term is negative; therefore, contrast is reduced. Similarly, taking \( \mu_2 > \mu_1 \) (i.e., the second object is ‘‘white’’), then \( \delta \) is negative and the second term is positive; therefore, the absolute value of the contrast is reduced. This simple analytical description is valid only for conditions where the two objects vary in linear attenuation coefficient by a small perturbation. The analytical form is nondivergent for \( \alpha > 0 \), and the range in \( \delta \) and \( \alpha \) for which the equation holds is set by the assumption that the presence of the second object significantly influences neither the measurement of \( \mu_1 \) nor the SPR.

### E. Effect of x-ray scatter on voxel noise

The noise in FPI-CBCT images was measured as a function of exposure and voxel size and compared to the analytical form derived by Barrett, Gordon, and Hershel.\(^{26} \) Voxel noise, \( \sigma_{\text{vox}} \), was determined as described previously\(^1 \) from the average of the standard deviations in circular realizations taken from transaxial slices in water (40 circular realizations at radii of 2–3 cm from the center of reconstruction). The effect of exposure was determined by measuring \( \sigma_{\text{vox}} \) for FPI-CBCT scans with total exposure in air ranging from \( \chi_{\text{iso}} \approx (1.3–11.3) \times 10^{-4} \) C/kg (i.e., \( \approx 0.5–4.4 \) R). The voxel noise in FPI-CBCT reconstructions has been shown previously to decrease in proportion to the inverse-square-root of exposure\(^{1,5} \) in agreement with the analytical description of Barrett, Gordon, and Hershel:\(^{26} \)

\[ \frac{\sigma_{\text{vox}}^2}{\mu^2} = \frac{kE_f e^{-\mu d}}{\rho \mu h \eta D_{\text{center}} a_{\text{res}}^3}. \tag{4} \]

where \( E_f \) is the x-ray energy (taken as the mean energy of the spectrum,\(^{32} \) 60 keV), \( f \) is a Compton factor (equal to the ratio of linear and energy attenuation coefficients), \( \mu \) is the average linear attenuation coefficient, \( d \) is the diameter of the cylinder, \( \rho \) is the density of water, \( \eta \) is the efficiency of detection (taken as the measured low-frequency detective quantum efficiency,\(^{1,5} \) \( 0.40 \)), \( h \) is the slice thickness, \( a_{\text{res}} \) is the transaxial resolution length, \( D_{\text{center}} \) is the dose to the center of the phantom (cGy), and \( k \) is a constant for proper units.

The factor \( I \) is related to the integral in the spatial-frequency domain of the ramp function and reconstruction filter (both squared),\(^{26} \) similar to the information bandwidth integral of Wagner, Brown, and Pastel.\(^{33} \) For full-resolution reconstruction (i.e., 0.25-mm cubic voxels), the slice thickness and resolution length were taken equal to the measured FWHM of the point-spread function,\(^1 \) 0.5 mm. To examine the effect of spatial resolution on voxel noise, the resolution length in the \( z \)-dimension \( (a_z) \) was varied from \( a_z = 0.5–3 \) mm, while the transaxial resolution length \( (a_x \) and \( a_y) \) was held constant at 0.5 mm. This examines one method (namely, slice averaging) by which reducing spatial resolution can reduce noise, neglecting the effects of noise aliasing.\(^{5,34} \) Variation of transaxial resolution length and reconstruction filter are subjects of future investigation.

Voxel noise was measured as a function of SPR by acquiring FPI-CBCT scans of the water cylinder at various settings of \( \phi_{\text{cone}} \), with phantom thickness fixed at \( T_{\text{PMMA}} \approx 30 \) cm. For a given tube output (i.e., mAs per projection) the exposure at the FPI, \( X_{\text{FPI}} \) increases with cone angle due to increased scatter. Therefore, the voxel noise is expected to decrease with increasing cone angle in a manner consistent with the relationship between noise and dose in Eq. (4).

### F. Effect of x-ray scatter on CNR

The contrast-to-noise ratio (CNR) was analyzed from volumetric images of the breast-equivalent phantom \( (i.e., \) in the breast-equivalent insert in water) to examine the degradation in image quality due to x-ray scatter and to determine the extent to which the CNR can be restored by increasing dose and/or reducing spatial resolution. As described above, the contrast was taken as the difference between the ensemble averages of voxels in the breast-equivalent insert and in water. The noise was taken as the average of the standard deviations in the ensembles and was consistent with measurements of the voxel noise in water described in the previous section. Taking the ratio of the contrast and noise, the CNR was analyzed as a function of SPR at three exposure levels: \( \chi_{\text{iso}} \approx 1.8 \times 10^{-4}, \) \( 2.6 \times 10^{-4}, \) and \( 6.2 \times 10^{-4} \) C/kg (i.e., \( 0.7, 1.0, \) and \( 2.4 \) R); and three values of \( z \)-dimension resolution length: \( a_z = 0.5, 1, \) and 2 mm, with \( a_x \) and \( a_y \) fixed at 0.5 mm. Results were compared to the analytic forms of Eqs. (3) and (4), from which we have:

\[ \frac{\alpha_{\text{vox}}^2 h}{I} D_{\text{center}} = \frac{\rho h \eta D_{\text{center}} a_{\text{res}}^3}{kE_f e^{-\mu d}} \left( \frac{1}{1+S/P} \right)^2, \tag{5a} \]

Rewriting the equation to express the spatial resolution and dose required to achieve a given CNR gives:

\[ \frac{\alpha_{\text{vox}}^3 h}{I} D_{\text{center}} = \frac{\rho h \eta D_{\text{center}} a_{\text{res}}^3}{kE_f e^{-\mu d}} \left( \frac{1}{1+S/P} \right)^2. \tag{5b} \]
dose and/or spatial resolution required to restore CNR to the value exhibited under scatter-free conditions. The approach is similar to that of Cohen,\textsuperscript{35} who considered the iso-noise relationship between dose and slice thickness as in Eq. (4) in examining the contrast-detail performance of slice-based CT scanners, and to Joseph,\textsuperscript{36} who examined the effect of image smoothing on low-contrast visualization. In examining the effects of x-ray scatter on contrast and noise in FPI-CBCT, we consider the iso-CNR relationships in Eq. (5b) as a function of SPR and compute the tradeoffs in dose and longitudinal resolution length in maintaining a given CNR. The parameters of transaxial resolution length ($a_{\text{res}}$) and reconstruction filter (contained in $I$) were not varied herein, but as discussed in Sec. III, certainly represent alternative mechanisms by which the CNR can be managed.

III. RESULTS

A. Magnitude of x-ray scatter

Figure 2(a) shows the SPR at the detector plane measured as a function of cone angle. The curves are linear fits to the data, showing that the slope (i.e., change in SPR per degree of cone angle) increases for thicker objects. All error bars herein indicate ±1 standard deviation from the mean. (b) Scatter fluence profiles at the detector plane for various settings of cone angle. For small cone angles, the scatter distributions are similar to those in slice-based CT, but increase significantly for larger cone angles.

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as a function of cone angle for five thicknesses of PMMA: 0 cm (“air”), 5 cm (“extremity”), 12 cm (“chest”), 18 cm (“abdomen”), and 30 cm (“pelvis”), for a fan angle of $\phi_{\text{fan}}=0.5^\circ$ (“low scatter”) or $\phi_{\text{fan}}=7^\circ$ (“high scatter”). All images herein are single transaxial slices from full-resolution FPI-CBCT reconstructions, and all image pairs [(a) and (b) throughout] were equivalently windowed and leveled for intercomparison.

FIG. 3. Transaxial images of a uniform water cylinder acquired under conditions of (a) low- and (b) high-scatter conditions. These and subsequent images were acquired with the phantom surrounded by PMMA of thickness $T_{\text{PMMA}}=30$ cm, with a fan angle of $\phi_{\text{fan}}=14^\circ$, and a cone angle of $\phi_{\text{cone}}=0.5^\circ$ (“low scatter”) or $\phi_{\text{cone}}=7^\circ$ (“high scatter”). All images herein are single transaxial slices from full-resolution FPI-CBCT reconstructions, and all image pairs [(a) and (b) throughout] were equivalently windowed and leveled for intercomparison.
expected for a given anatomical site and cone angle (e.g., SPR \(\sim\)87\% for the abdomen at \(\phi_{\text{cone}}=10^\circ\)). Measurements of SPR were also performed for the larger fan angle, \(\phi_{\text{fan}}\sim22^\circ\), corresponding to a large FPI (at least 64 cm in width) in the centered geometry. For the larger fan angle (\(\phi_{\text{fan}}\sim22^\circ\)), the SPR increases such that the slopes increase by a factor of \(\sim1.33\).

Figure 2(b) shows scatter fluence profiles measured behind the water cylinder at various settings of cone angle (and, therefore, various levels of SPR). For the smaller cone angles, the scatter profiles are lower in magnitude and exhibit a concave-upward shape similar to that calculated by Glover\(^{15}\) for conventional fan-beam CT, where the scatter distribution is reduced in the center due to self-attenuation of scattered photons. For larger \(\phi_{\text{cone}}\) (and, therefore, SPR), the scatter profiles increase in magnitude and exhibit an increasingly concave-downward shape due to a higher scatter contribution from out-of-plane.

Figure 4(a) shows scatter fluence profiles measured behind the water cylinder at various settings of cone angle and, therefore, various levels of SPR. The inaccuracy, \(\Delta\), defined as the percent deviation in mean reconstruction value from the expected value, is plotted versus SPR on the left-hand axis of (b). The nonuniformity, \(t_{\text{cup}}\), defined as the relative deviation between voxel values in the center of the reconstruction compared to those at the edge, is plotted on the right-hand axis of (b).

B. Shading artifacts and quantitative accuracy

FPI-CBCT images of the uniform water cylinder provided qualitative visualization and quantitative analysis of shading artifacts induced by x-ray scatter. For example, Fig. 3 shows transaxial images of the water cylinder acquired under conditions of low scatter (i.e., \(\phi_{\text{cone}}\sim0.5^\circ\) and \(T_{\text{PMMA}}=30\) cm, giving SPR \(\sim\)10\%) and high scatter (i.e., \(\phi_{\text{cone}}\sim7^\circ\) and \(T_{\text{PMMA}}=30\) cm, giving SPR \(\sim\)120\%). For the former, the image is uniform throughout the reconstructed volume, whereas the latter exhibits a pronounced nonuniformity in the form of reduced voxel values near the center of the image (i.e., a cup artifact\(^{14–16,23}\) common to conventional CT). The magnitude of the cup artifact is shown in Fig. 4(a), where diametric profiles are plotted for various values of SPR. For conditions of low scatter (e.g., SPR \(\sim\)2\%), the signal profiles are uniform within the cylinder and exhibit reconstruction values close to the expected value of 0.020 mm\(^{-1}\). As SPR...
increases, two effects are evident: an overall reduction (i.e., inaccuracy) in reconstruction values, and an increased severity (i.e., nonuniformity) of the cup artifact. These effects are quantified in Fig. 4(b), which plots the inaccuracy, $\Delta$, and nonuniformity, $t_{\text{cup}}$, as a function of SPR. For SPR in excess of $\sim 100\%$, average reconstruction values are inaccurate (i.e., underestimated) by more than 30%. The relative degree of nonuniformity in the image increases from $t_{\text{cup}} \sim 2\%$ for SPR $\sim 10\%$ to nearly 20% cupping for SPR in excess of $\sim 100\%$.

Images of the water cylinder containing two bone inserts are shown in Fig. 5 for conditions of low and high scatter (SPR $\sim 10\%$ and $\sim 120\%$, respectively), illustrating the additional shading artifact of a dark streak between the bones. For the lower-scatter case, the streak artifact is evident but fairly subtle, accentuated by a photon starvation artifact of smaller light and dark streaks between the bones. For the higher-scatter case, the streak artifact is prominent and dominates the underlying cup artifact. Moreover, note that the magnitude of voxel noise throughout the image as well as the severity of the photon starvation artifact appear to be reduced for the higher scatter case. This qualitative observation is consistent with the expectation mentioned above that for higher SPR, the exposure to the detector increases, and the voxel noise is reduced.

C. Contrast, noise, and CNR

The contrast of the breast-equivalent insert in water was measured as a function of SPR, as shown in Fig. 6. For the lowest scatter conditions, the measured contrast is $\sim 0.0009$ mm$^{-1}$ (i.e., $\mu_{\text{H}_2\text{O}}=0.0184$ mm$^{-1}$ and $\mu_{\text{breast}}=0.0175$ mm$^{-1}$), corresponding to a relative contrast of 5%. This value corresponds closely to the value expected based on manufacturer specifications of approximate CT# (i.e., $\text{CT}_{\text{H}_2\text{O}}=1000$ and $\text{CT}_{\text{breast}}=953.3$), giving relative contrast of 4.8%). As SPR increases, however, the measured contrast of the breast insert reduces significantly. For example, at SPR $\sim 100\%$ the contrast is reduced to $\sim 0.0004$ mm$^{-1}$ (i.e., 2.2% relative contrast). The curve in Fig. 6 represents the simple analytical form of Eq. (3d), which corresponds well with the measured results. The images in Fig. 7 illustrate the degradation in contrast with increased SPR. A previous study compared the image quality of the FPI-CBCT system with a conventional CT scanner for a low-contrast phantom containing the same breast insert under conditions of low x-ray scatter.

Contrast alone, however, does not determine the visibility of structures in tomographic images; rather, the visibility of large, low-contrast objects is affected by the contrast in proportion to the voxel noise, i.e., by the CNR. Figure 8 shows the voxel noise measured in FPI-CBCT reconstructions of
the uniform water cylinder as a function of exposure and SPR. Figure 8(a) shows that the voxel noise decreases with the inverse square root of exposure, in agreement with the analytic form of Eq. (4). Since the analytical relation involves measured input parameters such as efficiency, resolution length, and dose and assumes monoenergetic x-rays, the uncertainty in the theoretical curve is estimated to be no better than \( \pm 10\% \).

The solid symbols and solid curve in Fig. 8(a) correspond to volume reconstructions in which the resolution length is equal in all dimensions, i.e., \( a_x = a_y = a_z = 0.5 \) mm. Also shown are empirical and theoretical results in which the longitudinal resolution length (i.e., the slice thickness) is set to 1 mm (open circle with dotted curve) and 2 mm (open circle with dashed curve), relating the intuitive trend that reduction in spatial resolution reduces the voxel noise. Due to strong band-limiting of the projection noise-power spectra (e.g., by the x-ray converter and apodization filter) prior to back-projection and 3D sampling, the effects of 3D noise-power aliasing are small and are neglected in the present analysis.

Figure 8(b) shows the voxel noise measured as a function of SPR at various levels of x-ray tube output. In each case, the water cylinder is surrounded by PMMA of thickness \( T_{\text{PMMA}} = 30 \) cm, corresponding to the largest object examined ("pelvis") and the SPR was varied by adjusting the cone angle from \( 0.5^\circ \) to \( 10.5^\circ \). The three curves correspond to tube output levels of 1.5, 2.5, and 5.0 mAs per projection (i.e., \( X_{\text{iso}} = 2.3 \times 10^{-5}, 3.9 \times 10^{-5}, \) and \( 7.7 \times 10^{-6} \) C/kg, or \( 9, 15, \) and \( 30 \) mR, respectively). In each case, although the nominal tube output is fixed, the exposure at the detector...
increases with cone angle and SPR due to x-ray scatter, and the voxel noise is reduced accordingly. This is not to say that x-ray scatter is beneficial to image quality; rather, it reflects the fact that the detector does not distinguish between primary and scattered photons, and integrates incident quanta irrespective of their history. The effect is evident qualitatively in Figs. 3, 5, and 7, where the magnitude of voxel noise is slightly reduced in the higher SPR images.

Since for higher levels of SPR both the contrast and voxel noise are reduced, the effect of x-ray scatter on the CNR is something of a race between the degradation in contrast and the reduction in noise. The net effect is degradation in CNR, as qualitatively shown in the images of the breast insert in Fig. 7. The CNR was analyzed from such images as a function of SPR and compared to expectations based on the ratio of the analytical forms in Eqs. (3d) and (4). The solid circles and solid curve in Fig. 9(a) show the degradation in CNR with increased scatter for full-resolution reconstruction (i.e., \( a_x = a_y = a_z = 0.5 \text{ mm} \)), and the CNR falls by a factor of 2 across the range of SPR investigated.

Since the voxel noise improves for increased exposure and/or reduced spatial resolution [Eq. (4)], the CNR is expected to improve accordingly. For example, Fig. 9(a) illustrates the effect of reduced spatial resolution on the CNR, where the longitudinal resolution length (\( a_z \)) was varied from 0.5 to 2 mm. These results indicate that the CNR at a given level of SPR can be doubled by reducing the longitudinal spatial resolution by a factor of 4. Similarly, Fig. 9(b) illustrates the effect of increasing exposure (at full spatial resolution). The solid circles and solid curve are the same as in Fig. 9(a), while the open circles (dotted curve) and solid triangles (dashed curve) correspond to increased total scan exposure. The higher exposure measurements were not performed at SPR > -30% due to tube heating limitations at increased phantom thickness, but the overall trend is conveyed by the analytical curves. As expected, CNR improves with increased exposure.

A question arises, therefore, as to how the degradation in CNR incurred from elevated levels of SPR can be managed through knowledgeable selection of dose and spatial resolution. From Eq. (5b), which isolates these terms as a function of CNR, it is straightforward to compute the factor, called \( \alpha_D \), by which the dose must be increased (for a given spatial resolution) in order to restore the CNR to a certain level. Conversely, one can compute the spatial resolution (for a given dose) that results in the desired CNR. Conservatively, we take the value of the CNR in the scatter-free case as the desired value, and compute \( \alpha_D \) as a function of SPR for various settings of longitudinal resolution length, \( a_z \). Note that these calculations are conservative moreover in that they consider adjustment of spatial resolution through variation of \( a_z \) only. However, variation of the transaxial resolution lengths, \( a_x \) and \( a_y \), as well as the reconstruction filter (contained in the bandwidth integral, \( I \)) represent means of adjusting spatial resolution that are at least as viable. Since each of these parameters is selectable at the time of image reconstruction (within the constraints of 3D noise-power aliasing\(^6\)), voxel noise can effectively be tuned through combined adjustment of the resolution lengths (\( a_x, a_y, a_z \)) and reconstruction filter. Whatever the choice, the resulting spatial resolution must of course be consistent with the imaging task.

Results are shown in Fig. 10, where \( \alpha_D \) is computed for three settings of \( a_z \), providing quantitation of the dose-resolution tradeoffs in management of x-ray scatter and CNR. For SPR of 100%, for example, the top curve indicates that CNR can be restored to the level achieved in the scatter-free case (at full spatial resolution) by increasing dose by a factor of 4. Alternatively, the CNR can be restored by increasing both the dose and the longitudinal resolution length by a factor of 2. Finally, CNR can be restored without any increase in dose by increasing the longitudinal resolution length by a factor of 4. The decision as to which CNR restoration strategy is best (e.g., increasing dose or reducing spatial resolution) should, of course, be based on the clinical objective.

IV. DISCUSSION AND CONCLUSIONS

The magnitude of x-ray scatter with which flat-panel cone-beam CT must contend is high, even for systems employing large air gaps consistent with optimal imaging geometry.\(^6\) The analogy to projection imaging is obvious: just as 2D projection imagers must contend with higher levels of scatter than 1D linear scanning detectors, so must 3D volumetric imagers (e.g., FPI-CBCT) contend with higher...
levels than conventional tomographic imagers (e.g., slice-based CT). The benefits of volumetric imaging, however, warrant investigation of how best to reduce x-ray scatter and manage its deleterious effects. Strategies to be considered include the selection of the imaging geometry,\textsuperscript{5} the use of air gaps or scatter-rejection grids,\textsuperscript{24,25} the incorporation of scatter correction algorithms,\textsuperscript{22,23} and knowledgeable selection of acquisition and reconstruction parameters consistent with a given clinical objective.

Such considerations are relevant also in the development of helical CT scanners, e.g., multi-slice CT scanners,\textsuperscript{7} employing cone angles somewhat higher than traditional fan-beam systems. Although the cone angles for multi-slice CT systems are typically much smaller than envisioned for FPI-CBCT, even a slight increase in SPR at the detector can have significant deleterious effect. For example, Fig. 2(a) shows that for a cone angle of 1°, the SPR corresponding to imaging of large anatomy (e.g., the pelvis) is \(\sim 20\%\) for the described geometry. As shown in Fig. 9, this corresponds to \(\sim 15\%\) degradation in CNR compared to the scatter-free case.

The shading artifacts and quantitative inaccuracy in CT\# observed in FPI-CBCT are of similar form to (but of greater magnitude than) conventional, slice-based CT.\textsuperscript{14} It must be considered, however, that such artifacts and inaccuracy are only significant to the extent that they impede the clinical objective, and that the goal in FPI-CBCT imaging is not necessarily the same as with conventional CT. Specifically, the FPI-CBCT system being developed for therapy guidance\textsuperscript{1,11,12} need not replicate the image quality achieved by diagnostic imaging technology: rather, it should provide information that is of sufficient quality and geometric accuracy to confidently guide a given procedure. To the extent that artifacts and CT\# inaccuracy impede such a clinical objective, the use of correction algorithms or modified acquisition methods may be warranted.

High levels of x-ray scatter were found to significantly degrade the visualization of soft-tissue structures, causing a loss in CNR consistent with analytical descriptions of contrast\textsuperscript{19} and noise.\textsuperscript{26} This warrants careful consideration of how the effects of scatter can best be managed. First, it is clear from Fig. 2 that the cone angle should be kept as small as possible such that the volume of interest is still accommodated. For example, for FPI-CBCT imaging of the prostate (<8 cm in length), the cone angle should be limited to \(<5°\) in order to minimize the SPR at the detector. Second, a quantitative understanding of the relationship between CNR, dose, and spatial resolution allows knowledgeable management of x-ray scatter effects. For example, as shown in Fig. 10, the dose and spatial resolution can be tuned within the constraints of the clinical objective to obtain CNR equal to that achieved under conditions of low x-ray scatter. In conclusion, the magnitude and effects of x-ray scatter in FPI-CBCT are significant, but can be managed through knowledgeable selection of the imaging geometry, the image acquisition technique, and the image reconstruction parameters as dictated by the clinical objective.

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