Joint Estimation of Deformation and Penalized-Likelihood CT Reconstruction Using Previously Acquired Images

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Motivation

Sequential studies are common
- Diagnosis, treatment, follow-up
- Longitudinal studies, image-guided surgery, image-guided radiation therapy

Traditional acquisitions
- Neglect patient-specific information
- Treat each acquisition in isolation
- Utilize full dose protocol in each acquisition

Patient-specific Prior Information

Integrate patient-specific prior image in reconstruction
- Take advantage of huge amount of anatomical information
- Need to compensate for possible patient motion between acquisitions

Reduced data fidelity requirements
- Help solve ill-posed problem in reconstruction
- Enable dose reduction techniques
  - Sparse angular sampling / Reduced fluence
Penalized-Likelihood Framework

I. Penalized-Likelihood Estimator (PLE)

\[ \hat{\mu} = \arg \max_{\mu} \sum_{i=1}^{N} h_i \left( \left[ A \mu \right] \right) - \beta_p \left\| \Psi \mu \right\|_p^p \]

- Attenuation Estimate
- Data Fit Term
- Image Roughness Penalty

Modified $p$-norm: quadratic neighborhood near origin

II. Prior Image, Penalized-Likelihood Estimator (PIPLE)

\[ \hat{\mu} = \arg \max_{\mu} \sum_{i=1}^{N} h_i \left( \left[ A \mu \right] \right) - \beta_p \left\| \Psi_p \left( \mu - \mu_b \right) \right\|_p^p - \beta_s \left\| \Psi_s \mu \right\|_s^s \]

- Prior Image Penalty

III. Deformable Prior Image Registration, Penalized-Likelihood Estimator (dPIRPLE)

\[ \{ \hat{\mu}, \hat{\lambda} \} = \arg \max_{\mu, \lambda} \sum_{i=1}^{N} h_i \left( \left[ A \mu \right] \right) - \beta_p \left\| \Psi_p \left( \mu - W(\lambda) \mu_b \right) \right\|_p^p - \beta_s \left\| \Psi_s \mu \right\|_s^s \]

- Deformation Operator

Modified $p$-norm: quadratic neighborhood near origin

Stayman et al. 2011, Ding et al. 2012 (Rigid PIRPLE)
Joint Estimation

Estimation of attenuation and deformation parameters
Joint estimation to overcome limited accuracy in initial registration
Alternating maximization to optimize attenuation and deformation alternatively

Joint Estimation I: Image Update

Estimator reduced to standard PLE with prior image penalty

\[
\hat{\mu} = \arg \max_{\mu} \Phi(\mu, \lambda) = \arg \max_{\mu} \sum_{i=1}^{N_p} \left( \left| A_i \mu \right| \right) - \beta_p \sum_{i=1}^{N_p} \Psi_p \left( \mu - W(\lambda) \mu_p \right) - \beta_s \left| \Psi_{R}\mu \right|_s
\]

Separable Paraboloidal Surrogates (SPS) algorithm
Easily parallelizable for each voxel
Maintain monotonic convergence with surrogate for prior image penalty

\[
\hat{\beta} = \left[ \frac{\sum_{i=1}^{N_p} A_i h_i \left( \left| A_i \mu \right| \right) - \beta_p \sum_{i=1}^{N_p} \Psi_p \left( \mu - W(\lambda) \mu_p \right) - \beta_s \sum_{i=1}^{N_p} \left( \left| \Psi_{R}\mu \right|_s \right)}{\sum_{i=1}^{N_p} A_i c_i \left( \left| A_i \mu \right| \right) + \beta_p \sum_{i=1}^{N_p} \Psi_p \left( \mu - W(\lambda) \mu_p \right) + \beta_s \sum_{i=1}^{N_p} \left( \left| \Psi_{R}\mu \right|_s \right)} \right]
\]

\( f \) denotes the modified \( p \)-norm operator

Erdogan, Fessler et al. 1999
Joint Estimation II: Registration Update

Deformation operator
Cubic B-spline based Free Form Deformation (FFD)
Low dimension parameterization, $C^2$ continuity, multi-resolution

$$W_x(\lambda) = x + \sum_{x \in N_x} \lambda \beta \left( \frac{x - x_i}{\sigma} \right)$$

Estimator reduced to standard registration estimator

$$\hat{\lambda} = \arg \max_{\lambda} \Phi(\lambda; \mu) = \arg \max_{\lambda} \sum_{i \in I} h_i [(A\mu)] - \beta_p \left\| \Psi_p (\mu-W(\lambda) \mu_p) \right\|_{p_p}^p - \beta_m \left\| \Psi_m \mu_m \right\|_{m_m}^m$$

$$= \arg \min_{\lambda} \beta_p \left\| \Psi_p (\mu-W(\lambda) \mu_p) \right\|_{p_p}^p$$

Take advantage of existing package (e.g., Elastix)
Adaptive Stochastic Gradient Descent (ASGD)
Choose higher $p_p$ for better convergence (e.g. $p_p = 2$, SSD)

Alternating Maximization

Initialization
4 pyramids
1000 ASGD updates per pyramid
Joint Estimate

Deformation Parameters
$\lambda^0$ (Zero Displacement)

Attenuation Parameters
$\mu^0$ (PLE Reconstruction)

One Registration Update
$\lambda^{k+1}$
Fixed $\lambda^{k+1}$
One Image Update
$\mu^{k+1}$
Fixed $\mu^{k+1}$

$T$ times of SPS updates
Experimental Workflow

First scan
(Baseline Exam)  →  Petroleum Jelly Injection
(Tumor Growth)  →  Second scan
(Follow-up Exam)
**Experimental Workflow**

- **First scan** (Baseline Exam)
- Petroleum Jelly Injection (Tumor Growth)
- **Second scan** (Follow-up Exam)

**Follow-up scans:**
- 1.25 mAs/frame ("standard")
- 0.1 mAs/frame ("low exposure")
- Fully sampled (360 frames/360°)
- Sparse sampling (various/200°)

**Convergence**

Objective function difference versus iteration
RMSE difference versus iteration

Choose number of SPS updates per image update $T$

![Convergence Graphs](image)
Illustrating Joint Estimation

Residual Registration Error

Image Reconstruction

dPIRPLE Deformation Field

"True" Deformation Field

Matched

I: Sparse Sampling, Standard Exposure

360 Frames
1.25 mAs/Frame

Current Anatomy

FBP

PLE

PIPLE
dPIRPLE

20 Frames, 1.25 mAs/frame
I: Sparse Sampling, Standard Exposure

360 Frames
1.25 mAs/Frame

Current Anatomy

FBP

PLE

PIPLE

dPIRPLE

20 Frames, 1.25 mAs/Frame

II: Sparse Sampling, Low Exposure

{200, 100, 40, 20, 10} Frames
0.1 mAs/Frame

20 frames
(2 mAs)

10 frames
(1 mAs)
**II: Sparse Sampling, Low Exposure**

Structural Similarity (SSIM)

\[
SSIM(x, y) = \frac{l(x, y) \cdot c(x, y) \cdot s(x, y)}{L \cdot C \cdot S}
\]

Conclusions/Future Work

Motion compensation is essential for prior image utilization and false information prevention

Proposed joint estimation exhibits excellent performance and robustness against very high sparsity and low fluence in cadaver experiments

Future work

- Reduce residual registration error using more sophisticated registration methods
- Study metrics on prior image content in the reconstructed image
Penalty Strength Selection

Metric: Global RMSE
PLE: 1D search for $\beta_R$
PIPLE, dPIRPLE: 2D search for $\beta_R$ and $\beta_P$

Computation Time Comparison

Consider a maximum of 1000 SPS updates on 20 frames

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<th>Total Number of Registration Updates</th>
<th>Total Time for Registration Updates (min)</th>
<th>Total Time for 1000 SPS Updates (min)</th>
<th>Total Time for dPIRPLE (hr)</th>
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