Joint Estimation of Deformation and Penalized-Likelihood CT Reconstruction Using Previously Acquired Images

Hao Dang, Adam S. Wang, Zhe Zhao, Marc S. Sussman, Jeffreyle H. Siewerdsen, J. Webster Stayman*

Abstract—Patients often undergo repeated x-ray CT acquisitions in both diagnostic imaging (e.g., lung nodule surveillance) and image-guided interventions (e.g., biopsy needle guidance). Radiation dose is a particular concern in such sequential CT studies. Traditional imaging paradigms treat each acquisition in isolation, neglecting a wealth of patient-specific anatomical information from previous studies and failing to seize an opportunity for dose reduction and/or image quality improvement. We propose a reconstruction framework that incorporates a previously unregistered patient-specific prior image as part of a penalized-likelihood (PL) reconstruction. The method jointly estimates patient motion between the original acquisition and the current data, and the image attenuation parameters that are part of traditional reconstruction. Central to this approach is a deformable registration scheme that can accommodate motion – in the case of thoracic imaging, arising primarily from respiration and variations in patient setup. To investigate the performance of this approach, we performed cadaver experiments on a cone-beam CT test-bench simulating a lung nodule surveillance scenario with highly downsampled acquisitions (a factor of 18 exposure reduction). The proposed approach yields reconstructions with a major increase in image quality compared to traditional approaches as well as model-based approaches that neglect prior information or use an unregistered prior image, suggesting great potential for dose reduction while preserving image quality.

Index Terms—Deformable Motion Estimation, Prior Image Reconstruction, Penalized-likelihood Reconstruction

I. INTRODUCTION

Sequential CT acquisitions are conducted in many clinical scenarios. In diagnostic imaging, for example, lung nodule surveillance scans are used to assess tumor growth rates through estimation of doubling times or to monitor patients following therapy. In image-guided interventions, a pre-operative scan is typically used for surgical planning, and (perhaps multiple) intra-operative scans may be acquired for up-to-date visualization of tissue change or localization of surgical tools, e.g., guidance of a needle or other interventional device. Such imaging studies are traditionally formed through a series of complete acquisitions, making radiation dose a particular concern. Simply reducing the exposure per projection will increase noise in reconstructions, while reducing the number of projections makes the reconstruction problem more ill-posed, often resulting in significant artifacts and thereby reducing image quality. Thus, reconstruction methods that seek to preserve image quality in low-dose imaging should better utilize noisy measurements and/or incorporate additional information to overcome limited data.

*The authors are with Johns Hopkins University, Baltimore, MD 21212 USA (corresponding author is J. W. Stayman, phone: 410-955-1314; fax: 410-955-1115; e-mail: web.stayman@jhu.edu).

This work was supported in part by Varian Medical Systems, Inc.

Model-based iterative reconstruction (e.g., penalized-likelihood estimation) is one strategy that improves the utilization of noisy measurements. The general framework of the model-based approach permits incorporation of many aspects of the physical detection process (e.g., noise, arbitrary geometry, scatter, etc.). Moreover, the framework allows for various penalties for enforcing or encouraging desirable image properties to provide improved trade-offs between dose and image quality. However, such techniques rarely leverage the rich information found in sequential studies.

Despite the wealth of information that is shared in repeated acquisitions, traditional imaging paradigms treat each acquisition in isolation, neglecting previously measured patient-specific anatomical information (e.g., prior images) from prior acquisitions and an opportunity for dose reduction. The idea of incorporating prior images directly into reconstruction has been established in methods such as Prior Image Constrained Compressed Sensing (PICCS) techniques [1-2] in which an objective encouraging similarity with the prior image is posed, but sparse differences are allowed through the use of l1 norms. Data consistency is typically enforced through a linearized equality constraint, which may place limits on the complexity of the forward model and disregard noise. More recent PICCS techniques additionally incorporate a noise model [3].

We previously introduced a prior-image-based reconstruction approach [4] that combines both the likelihood-based framework and penalty functions that integrate prior-information. The approach allows for a great deal of flexibility in the sophistication of the forward model and noise model, since an explicit linearization of the model is not required.

A critical aspect of efficient use of previous anatomical information in any prior-image-based approach is the compensation of patient motion between acquisitions. Without motion compensation, prior-image-based reconstruction approaches cannot differentiate between true anatomical change (e.g., tumor growth or tissue resection) and change due to motion. Ambiguity between these two types of change makes true anatomical change difficult to recognize and could easily lead to introduction of false anatomical changes.

We choose to compensate patient motion by incorporating a proper 3D deformation model into a model-based iterative reconstruction method, and to use an optimization framework that jointly estimates deformation and the image reconstruction. The concept of joint estimation has been studied to recover periodic motion (e.g., cardiac and lung) from a single gated acquisition [5-7], but has had less attention within the context of prior-image-based reconstruction. In this work, we propose a framework that incorporates a patient-
specific prior image with a cubic B-spline based deformation model into penalized-likelihood estimation. An alternating maximization optimization was applied to solve the joint estimation problem, and the approach was evaluated in cadaver experiments emulating lung nodule imaging scenarios.

II. METHODS

A. Forward Model and Penalized-Likelihood Estimation

In x-ray CT, a discretized object $\mu$ can be related to mean measurements $\gamma$ via Beer’s Law, with a forward model written in matrix-vector form as

$$\gamma = D(g) \exp(-l), l = A \mu$$

where $D$ is an operator converting a vector of measurement-dependent gains $g$ to a diagonal matrix, $l$ denotes the vector of line integrals, and $A$ is the $M \times N$ system matrix (i.e., linear projection operator).

While one can apply an arbitrary noise model for the measurements, we choose a Poisson model with independent measurements $\gamma_i$ to yield the log-likelihood function in Eq. (2):

$$\log L(y; \mu) = \sum_i h_i \left(\left[\mu \alpha_i\right] - \sum_i \log \left(g \exp\left(-\left[\mu \alpha_i\right]\right)\right) - g \exp\left(-\left[\mu \alpha_i\right]\right)\right)$$

where $h_i$ denotes a marginal log-likelihood for each $i$.

A general penalized-likelihood estimation (PLE) can then be formed as the maximizer of the log-likelihood above with a general image regularizer $R(\mu)$, written as:

$$\mu = \arg \max_{\mu} \log L(y; \mu) - R(\mu), R(\mu) = \beta g^2 \| \Psi \Psi \|^2$$

A specific image roughness penalty has been chosen containing an image gradient operator $\Psi$, a p-norm, and a scalar control parameter $\beta$. A modified $p$-norm function has been implemented in which $p$ is the $\frac{1}{p}$-norm, $\frac{1}{p}$ is the gradient of (4) in the registration update is:

$$\frac{1}{p} \Delta^{(i)} = \arg \max_{\mu} \log L(y; \mu) - R(\mu), R(\mu) = \beta g^2 \| \Psi \Psi \|^2$$

where the additional prior penalty term contains its own image gradient operator $\Psi$, a p-norm, and a scalar $\beta$. The p-norm value $\beta$ can be freely chosen to achieve different performance. For example, $p=2$ tends to enforce smooth differences thereby blending features in the prior image and current measurements, while $p=1$ tends to encourage sparse differences (as one finds in compressed sensing).

The deformation $W(\mu)$ permits any suitable model. In this work, we adopted a cubic B-spline deformation model [8]:

$$W_\mu(x) = x + \sum_{i \in N_c} \lambda_i \beta(x - x_i)$$

where $\beta(\cdot)$ is the tensor product of cubic B-spline functions, $x_i$ are the control points, $\sigma$ is the control point spacing, $\lambda_i$ is the B-spline coefficient vectors (i.e., control point displacements), and $N_c$ is the set of control points within the support of the

B-spline at $x$. If $W(\lambda)$ is removed in (4), the prior image $\mu_p$ will not be motion compensated, referred to simply as Prior Image, Penalized Likelihood Estimation (PIPLE).

C. Optimization Algorithm for dPIRPLE

A modified $p$-norm function has been implemented in which the original p-norm in a $\delta$-neighborhood around the origin is replaced by a quadratic function so that the new function becomes differentiable at the origin. When $p=1$, the modified $p$-norm is equivalent to the sum of Huber loss functions evaluated at each element.

Although (4) becomes differentiable with a modified $p$-norm, it is still generally a nonconvex function of $(\mu, \lambda)$ and therefore challenging to optimize. The optimization can be simplified by using an alternating maximization approach in which we maximize over $\mu$ with fixed $\lambda$ and vice versa, i.e.:

$$\mu^{\text{opt}} = \arg \max_{\mu} \Phi(\mu, \lambda)$$

$$\lambda^{\text{opt}} = \arg \max_{\lambda} \Phi(\mu^{\text{opt}}, \lambda)$$

Image update: With fixed $\lambda$, (4) becomes a standard PLE with two penalty terms. This allows for the use of well-known optimization approaches for PLE. We choose the separable paraboloidal surrogates (SPS) approach [9], which makes the image update easily parallelizable for each voxel. Both penalty terms (for $p\geq 1$) meet the five criteria in [9] so that their surrogates can be found.

Registration update: With fixed $\mu$, (4) reduces to the prior image penalty term (dropping two constant terms) - i.e., the difference between the current estimate and the deformed prior image. This is essentially a standard image registration problem, with a modified $p$-norm as a similarity metric, which can be solved efficiently using existing registration software with minor modifications. For example, for $p=2$, this is equivalent to deformable registration with a Sum of Squared Differences (SSD) similarity metric. We choose to use a Limited-memory variant of BFGS (L-BFGS) [10] to approximate the Hessian matrix in our implementation. The gradient of (4) in the registration update is:

$$\frac{\partial}{\partial \lambda} \Phi(\mu, \lambda) = \frac{\partial}{\partial \lambda} \Phi(\mu, \lambda) = \left[ \Psi \Psi \right] (\mu - W(\lambda))$$

A pseudo-code representation of the optimization algorithm for dPIRPLE is in Algorithm 1.

Algorithm 1: dPIRPLE

Input $p^0$, $\lambda^0$, $\beta$, Inverse Hessian $H^{0,0}$, for $n = 0$ to max_iteration - 1
if do registration at iteration $n$
for $r = 1$ to $R$ % Registration update
  Compute gradient using (7), update $H^{r,0}$ from $H^{0,0}$, do line search with Wolfe condition, update $\lambda^{n+1}$ from $\lambda^{n,}\lambda^{n+1}$
end for
end if
for $j = 1$ to $N$ % Image update
  Update $\mu_j^{n+1}$ from $\mu_j^n$
end for
end for
return $\mu$ and $\lambda$. 
Fig. 1. (a) Cadaver and CBCT bench. (inset: petroleum jelly injector) (b) Patient-specific prior image, PLE of a complete pre-injection (nodule-absent) acquisition (360 views over 360°). (c) Current anatomy, PLE of a complete post-injection (nodule-present) acquisition. The nodule is marked by an arrow.

Fig. 2. (a-c) Difference between current anatomy and prior image before registration (a), after 1st registration update (b), and after 20th registration update (c). (d) Final deformation field estimated by dPIRPLE. (e) ‘Optimal’ deformation field acquired by two fully sampled datasets. Each vector represents the in-plane displacement of one voxel with vector magnitude scaled by a factor of 2 for visualization.

Whereas keeping the objective function strictly the same in two updates is preferred in solving a single objective function, in practice, one may choose different $p_P$ in the two updates. This can yield desirable convergence performance in both registration (where low $p$ values can lead to poor convergence) and the image update (which requires low $p$ values to encourage sparse differences). We implemented the dPIRPLE algorithm in Matlab, with computationally intensive functions (e.g., projection/backprojection) calculated using CUDA-based libraries and the registration toolbox Elastix. [11]

D. Cadaver Experimental Methods

We conducted cadaver experiments on a flat-panel cone-beam CT (CBCT) test-bench (Fig. 1(a)) to evaluate the dPIRPLE approach. The imaging task was to reconstruct a newly formed lung nodule (introduced between scans) in the presence of deformable patient motion between two acquisitions. The detector (PaxScan 4343CB, Varian Medical Systems, Palo Alto, CA) had 1536×1536 pixels at 0.278×0.278 mm² pixel pitch after 2×2 binning. The system geometry involved a 150 cm source-to-detector distance and 120 cm source-to-axis distance. All data were reconstructed with 260×300×330 voxels and 1×1×1 mm³ voxel size.

To simulate tumor growth between acquisitions, ~1 cm³ Petroleum jelly (~0.013 mm⁻¹ attenuation) was injected into the right lung of the cadaver by a thoracic surgeon. Two fully sampled datasets (360 views over 360° at 100 kVp and 450 mAs) were acquired before and after the injection. PLE ($p_R=2$, $\beta_R=10^3$) was applied to both the nodule-absent and nodule-present datasets to obtain the patient-specific prior image and current anatomy, respectively (Fig. 1(b-c)). Motion imparted between scans is evident in the difference image in Fig. 2(a). From the post-injection data, 20 projections equally spaced over 200° (25 mAs) were selected to simulate an undersampled low-dose follow-up lung surveillance image at 1/18 the exposure of a fully sampled acquisition.

III. EXPERIMENTAL RESULTS

We reconstructed highly undersampled projections using FBP (filtered-backprojection), PLE (FBP initialization, $p_R=1$, $\beta_R=10$), PIPLE (PLE initialization, $p_P=1$, $\beta_P=10^4$, $p_R=1$, $\beta_R=10^3$), and the proposed dPIRPLE (same parameters as PIPLE but with $p_P=2$ in registration update). In dPIRPLE, each registration update consisted of four levels of image pyramids (10×10×10 mm³ grid size at the finest level) followed by 50 image updates. We choose $\Psi_R$ equal to identity, $\Psi_P$ as the first-order spatial difference operator, and $\delta=10^{-4}$ mm⁻¹ in the modified $p$-norm. The total run time for 50 iterations was about 4 min for PLE, 7.5 min for PIPLE, and 14.5 min for dPIRPLE on a high performance workstation.

A. Convergence

Fig. 2 (a-c) shows the evolution of the deformation estimate versus iteration number. Specifically, we show the difference between the deformed prior image and current anatomy at different stages. We observe that most motion was compensated after the 1st registration update, which substantially prevented incorrect structures from being injected into subsequent image updates. The remaining differences (arrows in Fig. 2(b-c)) continued to be reduced after 20 updates, demonstrating the importance of joint estimation. The final deformation field estimated by dPIRPLE (Fig. 2(d)) closely matches an ‘optimal’ deformation field derived from

Fig. 3. (a) dPIRPLE objective function difference versus iteration. (b) Root Mean Square (RMS) difference from current anatomy versus iteration for PLE, PIPLE, and dPIRPLE. The asterisk indicates Iteration 0 outside the plot.
the fully sampled reconstructions (Fig. 2(e)). We computed root mean square (RMS) differences between the current anatomy and the deformed prior image in a Region of Interest (ROI, 100×100×100 voxels centered on the nodule): 76.5×10^{-4} mm^{-1} (at iteration zero), 24.6×10^{-4} mm^{-1} (after 20 registration updates), and 22.6×10^{-4} mm^{-1} (with ‘optimal’ registration).

Fig. 3(a) shows the objective function difference for dPIRPLE as a function of iteration. The objective function value at the solution, \( \Phi^* \), is estimated using 1000 iterations of dPIRPLE. The objective increases monotonically within every 50 image updates due to the monotonicity of SPS. The 1st registration yields a dramatic objective increase with smaller increases in subsequent registrations, consistent with Fig. 2. In plots of RMS difference from current anatomy versus iteration (Fig. 3(b)), PLE quickly reduced RMS difference, but plateaued. In contrast, dPIRPLE saw reduction throughout, and reduced 2× faster than PIPLE.

B. Reconstruction

Fig. 4 compares the reconstruction results from different approaches. We ran 1000 iterations of PLE, PIPLE, and dPIRPLE to generate (nearly) converged images. While FBP exhibits substantial artifacts that would confound nodule detection, PLE has reduced artifact but low spatial resolution owing to strong regularization and rendering makes the nodule difficult to detect. PIPLE exhibits higher contrast in the nodule area, but severe mismatches due to lack of proper registration distorts the nodule shape and introduces a considerable number of ambiguous structures which do not reflect true anatomy. Finally, dPIRPLE presents an accurate estimate of the true anatomy and renders the nodule clearly. The RMS difference from current anatomy within the ROI was 36.8×10^{-4} mm^{-1}, 31.1×10^{-4} mm^{-1}, and 31.9×10^{-4} mm^{-1} for FBP, PLE, and PIPLE, respectively, and 16.3×10^{-4} mm^{-1} for dPIRPLE.

IV. CONCLUSION

The proposed dPIRPLE approach demonstrates a major improvement in image quality under conditions of highly undersampled data compared to traditional approaches and model-based approaches that neglect prior information or use an unregistered prior image. This suggests that the dPIRPLE approach could be valuable in clinical scenarios offering a patient-specific prior and requiring dose reduction without loss in image quality. The dPIRPLE approach also shows that the joint maximization estimates patient motion more accurately than staged registration and yields an improved representation of true anatomy without false structures arising from misregistration. A limitation of the current work includes a residual registration error, which we plan to reduce using more sophisticated registration techniques, such as [12].

REFERENCES